# Rotary Actuators





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| | |
|---|---|
| A | Area |
| d | Diameter |
| F | Force |
| $g_c$ | Gravitational constant (386 in/sec$^2$) |
| I | Mass mount of inertia |
| m | Mass |
| P | Pressure |
| $P_C$ | Average cushion pressure |
| $P_r$ | Relief valve pressure |
| Q | Volumetric flow rate |
| r | Radius |
| $r_b$ | Bearing radius |
| t | Time |
| T | Torque |
| $T_\alpha$ | Angular acceleration torque |
| $T_{\alpha*}$ | Angular deceleration torque |
| $T_C$ | Cushion torque |
| $T_D$ | Demand torque |
| $T_f$ | Friction torque |
| V | Volume displacement |
| $V_s$ | Specific Volume in terms of in$^3$ per radian |
| W | Weight |
| x | Distance or position |
| $\alpha$ | Angular acceleration |
| $\mu$ | Coefficient of friction |
| $\theta$ | Angular displacement or rotation |
| $\omega$ | Angular velocity |
| $\omega_o$ | Angular velocity at time = o |
| $\omega_t$ | Angular velocity at time = t |

# Rotary Actuators

## Introduction

A rotary actuator is the most compact device available for producing torque from hydraulic or pneumatic pressure. A self-contained unit, it is usually limited to one revolution or less and can provide oscillating motion as well as high and constant torque. Figure 1 shows the standard symbol for pneumatic and hydraulic rotary actuators.

There are many types of rotary actuators, each with design advantages as well as compromises. The three most commonly used are rack and pinion, vane, and helical. These type actuators are compared in Table 3 on page 7.

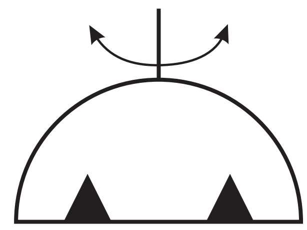

*Figure 1 Hydraulic rotary actuator symbol*

## Types of Rotary Actuators

### Rack and Pinion

Rack and pinion actuators consist of a housing to support a pinion which is driven by a rack with cylinder pistons on the ends (see Fig. 2). Theoretical torque output T, is the product of the cylinder piston area A, operating pressure P, and the pitch radius of the pinion $r_p$.

Equation 1) $T = PAr_p$

Single, double, or multiple rack designs are possible and overall efficiencies for rack and pinion units average 85 to 90%. Because standard cylinder components can be used to drive the rack, many standard cylinder features can be incorporated into rack and pinion actuators, such as cushions, stroke adjusters, proximity switches, and special porting. Additionally, virtually leakproof seals will allow the actuator to be held in any position under load.

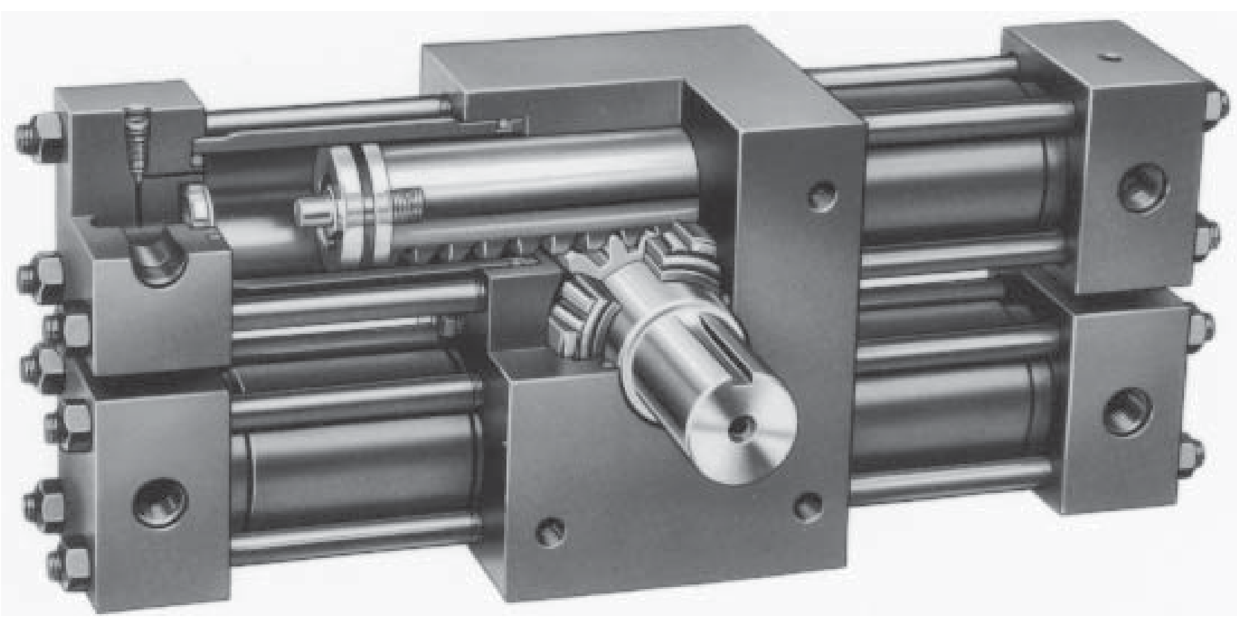

*Figure 2 Rack and pinion type rotary actuator, double rack design*

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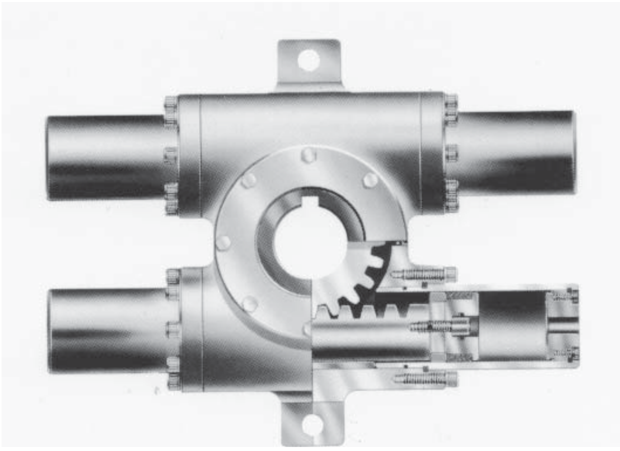


*Figure 3 Mill type actuator design*

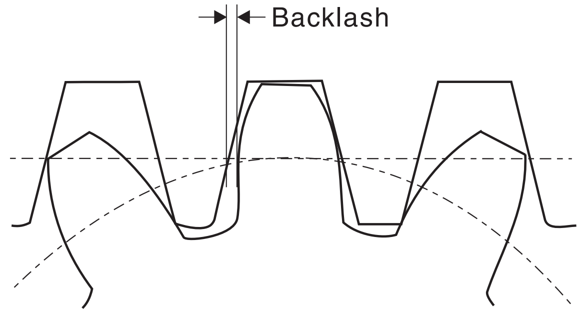



*Figure 4 Gear backlash affects position accuracy*

**NOTE: Some systems require a mechanical locking device for safety reasons, or for holding loads over extended periods of time.**

Both tie-rod and mill cylinder type (Fig. 3) constructions are available, and most types allow for service of all pressure containing seals without removing the unit from its mounting.

Rack and pinion actuators cover the widest range of torque, from 0.9 N•m *(8 lb. in.)* pneumatic to over 5,630,630 N•m *(50,000,000 lb. in.)* in hydraulic units. Because of their construction, they are not limited to 0.2π rad *(360 degrees)* of rotation and can easily be built to 10π rad *(1800 degrees)* (five revolutions). The majority of rack and pinion style actuators are sold for hydraulic service, generally 34 to 207 bar *(500 to 3000 psi)*.

Position repeatability in rack and pinion actuators is affected by the inherent backlash found in any gear arrangement. Backlash is the amount by which the width of a tooth space exceeds the thickness of the mating tooth and can be as much as π/360 *(0.5 degrees)* on smaller size units (Fig. 4). It should be noted, however, that this backlash can be reduced to almost nothing by preloading the rack into the pinion, but efficiencies will suffer to overcome the added friction.

Because the load ratings of the bearings used to support the pinion are large in comparison to the internal loading of the unit, extra bearing capacity is usually available. This can eliminate the need for machine support bearings, or handle overhung and thrust loads which would be detrimental to other types of rotary actuators. In other applications, a hollow pinion is used, which eliminates the need for a coupling and support brackets because the actuator can be mounted directly onto the input shaft.

## Table 1: Rotary Actuator Applications

### General Industry
Camming, indexing, clamping, positioning, tensioning, braking, tilting, etc.

### Material Handling
Switching conveyors, turning and positioning container clamps on lift trucks, tensioning and guiding, operating valves, braking, lifting

### Robotics
Rotation and positioning

### Marine
Opening and closing hatches, swinging cargo handling gear, opening and closing fire and collision bulkhead doors, operating large valves of all types, positioning hydrofoils, steering control

### Steel
Upending coils, turnstiles, rollover devices, tilting electric furnaces, indexing transfer tables, charging furnaces

In general, rack and pinion actuators have a thin profile, but are not as physically compact as other styles of rotary actuators, and are usually slightly more expensive than vane actuators of equivalent torque output.

## Vane

Vane style actuators consist of one or two vanes attached to a shaft (called the rotor), which is assembled into a body, and then held in place by two heads (Fig. 5). Rotation of single vane units is generally limited to 4.9 rad (280 degrees) by a fluid barrier (called a stator). Double vane units are limited to 1.7 rad (100 degrees) because two stators are required at opposite ends. The operating medium (air or oil) is ported across the shaft in double vane style actuators to eliminate the need for four ports. Fluid pressure acting on the exposed vane surface produces an output torque, shown in Equation 2:

Equation 2) $T = LWP$

Where the torque T is equal to the product of the vane length L, times the vane width W, times the system pressure P, times the radial distance r from the center of the rotor to the vane pressure center. Of course, a double vane style actuator will have twice the area of a single vane style actuator, and therefore twice the torque.

Available industry sizes range from a minimum of 1.4 N•m *(12 lb. in.)* with small pneumatic actuators to a maximum of 84460 N•m *(750,000 lb. in.)* with high pressure hydraulic actuators.

Vane style actuators have no backlash, but because of the seal configuration, cannot hold position without pressure being applied. The vane seal typically has sharp corners to seal in the body/head interface. Since this corner cannot be sealed completely, there is always a slight bypass flow. There is additional bypass flow in the shoulder area of the vane, so even rounding the vane at the top does not completely eliminate leakage. Vane actuators require external stops to prevent damage to the vane and stator, especially for high inertia and high speed applications.

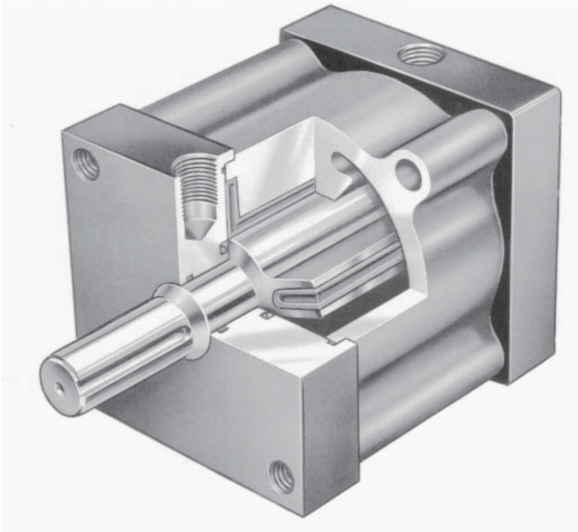


*Figure 5 Vane type rotary actuaor*

Vane actuators are very compact devices and are typically less expensive than rack and pinion units of equivalent torque and pressure. Units are available with and without external shaft load capacity.

Vane actuators can be segmented into three general product lines:

1) Small pneumatic actuators for small parts handling, fixturing, etc.
2) 69 bar *(1000 psi)* hydraulic actuators for machine tool, automotive equipment, and transfer lines.
3) 207 bar *(3000 psi)* actuators for the mobile equipment industry.

## Helical

Helical actuators consist of a piston sleeve, which functions similarly to a cylinder piston, and a rotating output shaft encased in a cylinder type housing (See Fig. 7). The linear motion of the piston sleeve produces rotary motion of the output shaft through the male helix cut on the shaft and a fixed helical nut. The torque output is proportional to the helix angle, system pressure, piston area, and the mean pitch radius of the helical shaft.

Helix designs provide maximum torque output for the smallest possible cross section. Double helix designs are also available to reduce the length of the unit or double the torque output.

Helical units are generally the most expensive rotary actuators but also the most compact. They do have backlash and can be made self-locking with special helix designs. Helical units can be hydraulically or pneumatically operated and are available from 2.3 to 450,451 N•m *(20 lb. in. to 4,000,000 lb. in.)* of torque.

### Helical Type Actuators

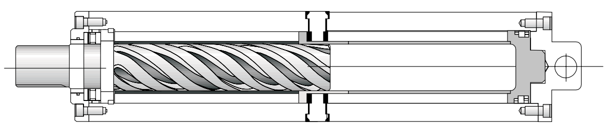


*Figure 7a Initial position*

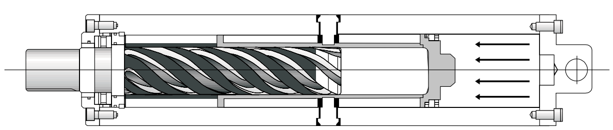


*Figure 7b Pressure moves the sleeve causing the helix to rotate.*

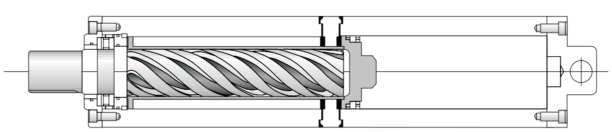


*Figure 7c Final position*

## Table 2: Rotary Actuator Comparison Chart

| Feature | Rack & Pinion | Vane | Helical |
|---|---|---|---|
| Load holding ability | Leakproof cylinder seals allow holding of load in any position. | Square vane seals and shoulder seals always have slight bypass flow. | Leakproof cylinder seals will allow holding of load in any position. |
| Positioning | Inherent backlash of rack and pinion cause position tolerance up to ½°. | Zero backlash allows for exact positioning anywhere in the rotation. | Some backlash, but can be made locking with special helix designs. |
| Efficiency (hydraulic) | 90% is average | 75-80% is average | 60% is average |
| Stops | External stops usually not required. | Internal stops available for some light duty applications; majority of applications require positive external stops. | External stops usually not required. |
| Cushions | Standard cylinder cushions be used. | Typically no cushions available. Parker HRN Series offers these. | Consult manufacturer |
| Size | Thin profile but larger overall space and weight requirements | Very compact, especially cross section | The most compact cross section for a given torque |
| Maintenance | Maintenance of pressure seals possible without complete disassembly of unit. | Maintenance of vane requires disassembly of unit. | Maintenance of seals requires disassembly of unit. |
| Mounting | Mounting styles include lug, foot, face, base, flange or shaft mounting with hollow pinion. | Mounting styles include base, foot, face or flange mounting. | Mounting styles include foot, flange or body mounting. |
| Operating Medium | Air or hydraulic operation | Air or hydraulic operation | Air or hydraulic operation |
| Available Rotation | 90°, 180° and 360° standard, specials to 1800° | 280° maximum single vane unit, 100° maximum double vane unit | Consult manufacturer; specials made to order |
| Price | Generally more expensive than equivalent torque vane units | Generally less expensive than equivalent torque rack and pinion units | Generally much more expensive than equivalent rack and pinion units |

## Design torque

Design torque represents the maximum torque that an actuator must supply in an application. This maximum is the greater of the Demand Torque or the Cushion Torque. If the demand torque exceeds what the actuator can supply, the actuator will either move too slowly or stall. If the cushion torque is too high, the actuator may be damaged by excessive pressure. Demand torque and cushion torque are defined below in terms of load, friction, and acceleration torque.

Equations for calculating demand torque and cushion torque for some general applications are provided on the following pages.

## T - Torque

The amount of turning effort exerted by a rotary actuator.

## $T_D$ - Demand Torque

This is the torque required from the actuator to do the job and is the sum of the load torque, friction torque, and acceleration torque, multiplied by an appropriate design factor. Design factors vary with the applications and the designers' knowledge.

Equation 3) $$T_D = T_\alpha + T_f + T_L$$

## $T_L$ - Load torque

This is the torque required to equal the weight or force of the load. For example, in Fig. 8a, the load torque is 563 N•M *(5000 lb-in.)*; in Fig. 8b the load torque is zero; in Fig. 8c the load torque is 563 N•m *(5000 lb-in.)*. The load torque term is intended to encompass all torque components that aren't included in the friction or acceleration terms.

## $T_f$ - Friction torque

This is the torque required to overcome friction between any moving parts, especially bearing surfaces. In Fig. 8a, the friction torque is zero for the hanging load; in Fig. 8b the friction torque is 775 N•m *(6880 lb-in)* for the sliding load; in Fig. 8c the friction torque is zero for the clamp.

Equation 4) $$T_f = \mu W r$$

## $T_\alpha$ - Acceleration Torque

This is the torque required to overcome the inertia of the load in order to provide a required acceleration or deceleration. In Fig. 8a the load is suspended motionless so there is no acceleration. In Fig. 8b, the load is accelerated from 0 to some specified angular velocity. If the mass moment of inertia about the axis of rotation is I and the angular acceleration is $\alpha$, the acceleration torque is equal to $I\alpha$. In Fig. 8c there is no acceleration.

Some values for mass moment of inertia are given in Table 3. Some useful equations for determining $\alpha$ are listed in Table 5. Equation 5 below shows the general equation for acceleration torque.

Equation 5) $$T_\alpha = I\alpha$$

## $T_C$ - Cushion Torque

This is the torque that the actuator must apply to provide a required deceleration. This torque is generated by restricting the flow out of the actuator (meter-out) so as to create a back pressure which decelerates the load. This back pressure (deceleration) often must overcome both the inertia of the load and the driving pressure (system pressure) from the pump. See applications.

Equation 6) $$T_C = T_{\alpha*} + \frac{P_r V}{\theta} - T_f \pm T_L$$

The friction torque $T_f$ reduces the torque the actuator must apply to stop the load. The load torque $T_L$ may add to, or subtract from the torque required from the actuator, depending upon the orientation of the load torque. For example, a weight being swung upward would result in a load torque that is subtracted.

**Warning: Rapid deceleration can cause high pressure intensification at the outlet of the actuator. Always insure that cushion pressure does not exceed the manufacturer's pressure rating for the actuator.**

# Demand Torque Examples

**A) Due to load torque**
The load is held motionless as shown.

$T_D = T_\alpha + T_f + T_L$

$T_\alpha = 0$

$T_f = 0$

$T_L = (500 \text{ lb})(10 \text{ in}) = 5{,}000 \text{ lb-in}$

$T_D = 5{,}000 \text{ lb-in}$

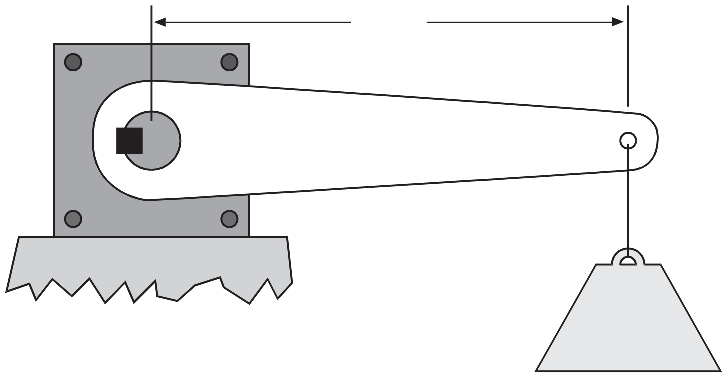


*Figure 8a*

**B) Due to friction and acceleration**
The 500 lb rotating index table is supported by bearings with a coefficient of friction of .25. The table's acceleration $\alpha$ is 2 rad/sec$^2$. The table's mass moment of inertia I is 2,330 lb-in-sec$^2$.

$T_D = T_\alpha + T_f + T_L$

$T_\alpha = I\alpha = (2{,}330 \text{ lb-in-sec}^2)(2/\text{sec}^2) = 4{,}660 \text{ lb-in}$

$T_f = \mu W r_b = 0.25\ (500 \text{ lb})(55 \text{ in}) = 6{,}880 \text{ lb-in}$

$T_L = 0$

$T_D = 4{,}660 \text{ lb-in} + 6{,}880 \text{ lb-in} = 11{,}540 \text{ lb-in}$

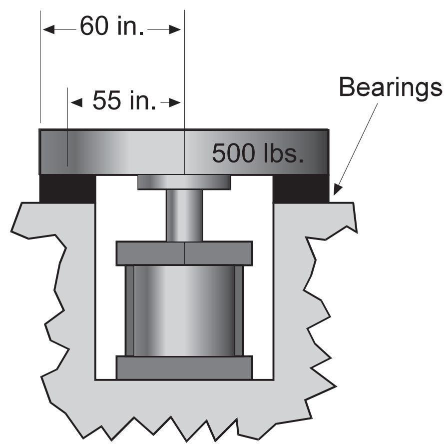



*Figure 8b*

**C) Due to load torque**

$T_D = T_\alpha + T_f + T_L$

$T_\alpha = 0$

$T_f = 0$

$T_L = (500 \text{ lb})(10 \text{ in}) = 5{,}000 \text{ lb-in}$

$T_D = 5{,}000 \text{ lb-in}$

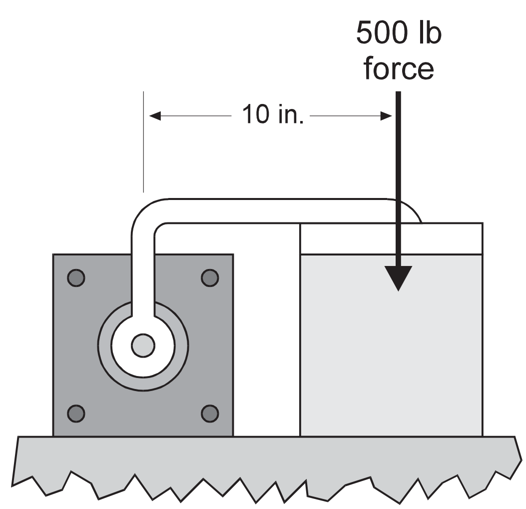


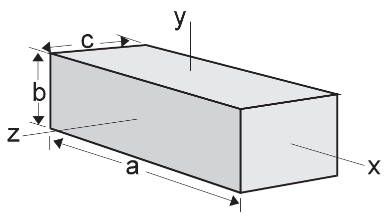

*Figure 8c*

Engineering

## Table 3: Mass Moments of Inertia

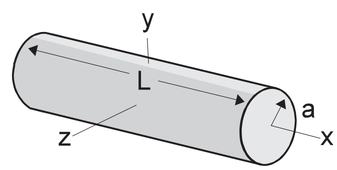
### Rectangular prism

$$I_x = \frac{1}{12}m\left(b^2 + c^2\right)$$
$$I_y = \frac{1}{12}m\left(c^2 + a^2\right)$$
$$I_z = \frac{1}{12}m\left(a^2 + b^2\right)$$



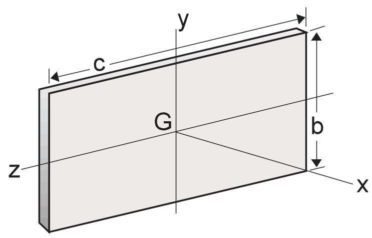

### Circular cylinder

$$I_x = \frac{1}{2}ma^2$$
$$I_y = I_z = \frac{1}{12}m\left(3a^2 + L^2\right)$$



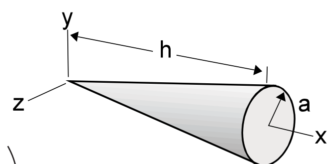

### Thin rectangular plate

$$I_x = \frac{1}{12}m\left(b^2 + c^2\right)$$
$$I_y = \frac{1}{12}mc^2$$
$$I_z = \frac{1}{12}mb^2$$



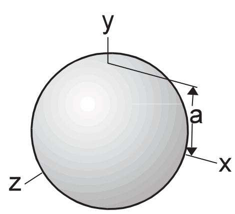

### Circular cone

$$I_x = \frac{3}{10}ma^2$$
$$I_y = I_z = \frac{3}{5}m\left(\frac{1}{4}a^2 + h^2\right)$$



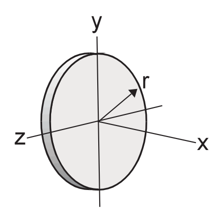

### Sphere

$$I_x = I_y = I_z = \frac{2}{5}ma^2$$



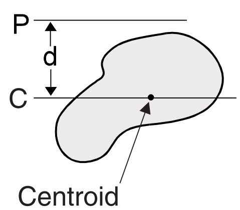

### Thin disk

$$I_x = \frac{1}{2}mr^2$$
$$I_y = I_z = \frac{1}{4}mr^2$$




### Parallel Axis Theorem:

$$I_p = \bar{I} + md^2$$




$I_p$ = Mass moment of inertia about an axis parallel to a centroidal axis.
$\bar{I}_C$ = Mass moment of inertia about a centrodidal axis.
m = Mass
d = distance between axes

### When acceleration is constant:

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\theta = \omega_0 t + \frac{1}{2}\omega_t t$$
$$\omega = \omega_0 + \alpha t$$
$$\omega = \left(\omega_0^2 + 2\alpha\theta\right)^{1/2}$$

$$\alpha = \frac{2\theta}{t^2}$$
$$\alpha = \frac{(\omega_t - \omega_0)^2}{2\theta}$$
$$\alpha = \frac{(\omega_t - \omega_0)}{t}$$

### When velocity is constant:

$$\theta = \omega t$$

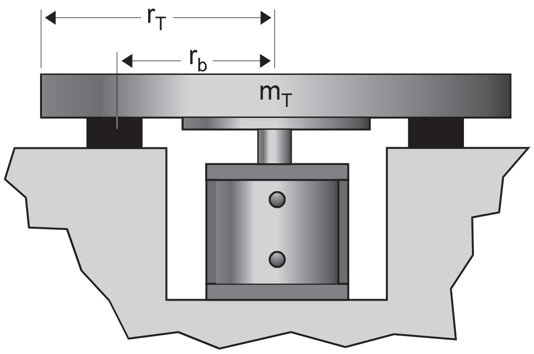
t = time
$\theta$ = angular distance
$\omega_t$ = angular velocity at time = t
$\omega_0$ = angular velocity at time = 0
$\alpha$ = angular acceleration

**NOTES:**

1. The following equations are intended only as a guide. The design engineer should verify the accuracy of the equations and insure that all performance, safety and warning requirements of the application are met.
2. Unless specified otherwise, the following examples do not take into account actuator efficiency, system efficiency, friction or needed safety factors.
3. Deceleration torques are based upon the assumption that restrictor type flow controls force the pump to exert maximum pressure during deceleration. This is denoted as $P_r$.
4. Symbol followed by * indicates a deceleration.

## Round Index Table

A round index table that must overcome bearing friction and inertia. Assumptions, see notes 1, 2, and 3.




$$T_D = \frac{1}{2} m_1 r_1^2 \alpha + \mu W_1 r_b$$

$$T_C = \frac{1}{2} m_1 r_1^2 \alpha + I_1 \alpha$$

$$T_D = T_\alpha + T_f + T_L$$

$$T_L = 0\text{, no load}$$

$$T_f = \mu W_1 r_b\text{, where } W_1 = m_1 g_C$$

$$T_\alpha = I_1 \alpha$$

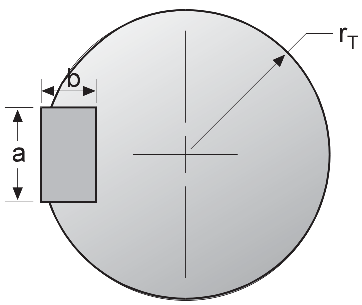
$$I_1 = \frac{1}{2} m_1 r_1^2$$

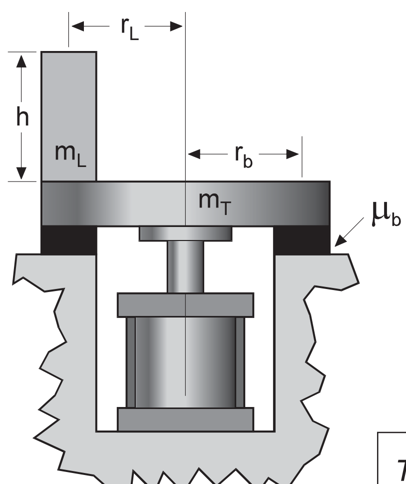
$$T_C = \frac{P_r V}{\theta} + T_{\alpha^*} - T_f \pm T_L$$

$$P_C = T_C \left[\frac{\theta}{V}\right]$$

## Rotay Index Table with Rectangular Load

An index table rotating in a horizontal plane with a rectangular box. It must overcome bearing friction and inertia. Assumptions, see notes 1, 2, and 3.




$$T_D = T_\alpha + T_f + T_L$$

$$T_L = 0\text{, no load}$$

$$T_f = (W_T + W_L) r_b \mu_b,$$

$$\text{where } W_T = m_T g_C,\ W_L = m_L g_C$$

$$T_\alpha = (I_T + I_L)\alpha$$

$$I_T = \frac{1}{2} m_T r_T^2$$

$$I_L = \frac{1}{12} m_L (a^2 + b^2) + m_L r_L^2$$

$$T_C = \frac{P_r V}{\theta} + T_{\alpha^*} - T_f \pm T_L$$

$$T_{\alpha^*} = (I_T + I_L)\alpha^*$$

$$P_C = T_C \left[\frac{\theta}{V}\right]$$

$$T_D = (W_T + W_L) r_b \mu_b + \alpha \left[\frac{1}{2} m_T r_T^2 + \frac{1}{12} m_L (a^2 + b^2) + m_L r_L^2\right]$$

$$T_C = (\alpha + \alpha^*) \left[\frac{1}{2} m_T r_T^2 + \frac{1}{12} m_L (a^2 + b^2) + m_L r_L^2\right]$$

## Rotary Index Table with Cylindrical Load

An index table rotating in a horizontal plane with a cylindrical load. It must overcome bearing friction and inertia. Assumptions, see notes 1, 2, and 3 on page 4-12.

$$T_D = T_\alpha + T_f + T_L$$

$$T_L = 0\text{, no load}$$

$$T_f = (W_T + W_L) r_b \mu_b,$$

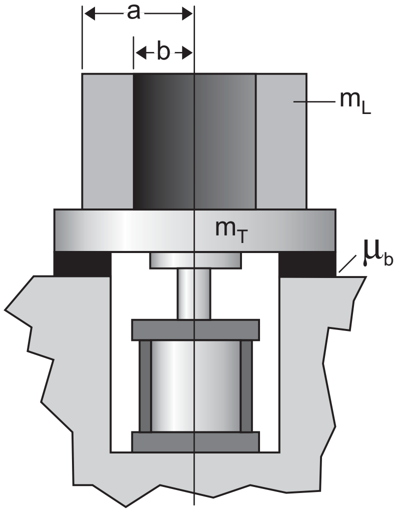
$$T_\alpha = (I_T + I_L)\alpha$$

$$I_T = \frac{1}{2} m_T r_T^2$$

$$I_L = \frac{1}{2} m_L (a^2 - b^2)$$

$$T_C = \frac{P_r V}{\theta} + T_{\alpha^*} - T_f \pm T_L \qquad P_C = T_C \left[\frac{\theta}{V}\right]$$

$$T_\alpha{}^* = (I_T + I_L)\alpha^*$$




$$T_D = (W_T + W_L) r_b \mu_b + \frac{\alpha}{2} \left[m_T r_T^2 + m_L (a^2 - b^2)\right]$$

$$T_C = \frac{1}{2} (\alpha + \alpha^*) \left[m_T r_T^2 + m_L (a^2 - b^2)\right]$$

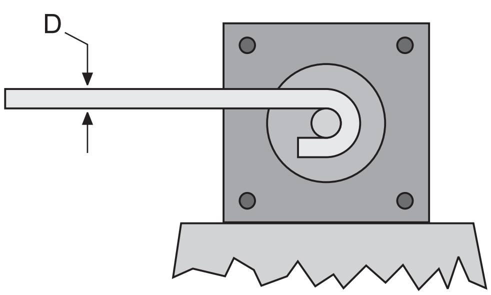
## Wire or Round Tube Bending

Bending tube or wire, no acceleration or friction concerns.

No acceleration, $T_\alpha = 0$
No friction, $T_f = 0$

$$T_D = T_L$$

$$T_L = \sigma_y \frac{I}{c}$$




Round tubing: $$T_D = \frac{\pi}{32} \left(\frac{D^4 - d^4}{D}\right) \sigma_y$$

Round wire: $$T_D = \frac{\pi \sigma_y D^3}{32}$$

$\sigma_y$ = yield stress of the material (available from mechanical engineering texts)

$\frac{I}{c}$ = section modulus of the tube or wire (can be calculated or found in materials handbooks)

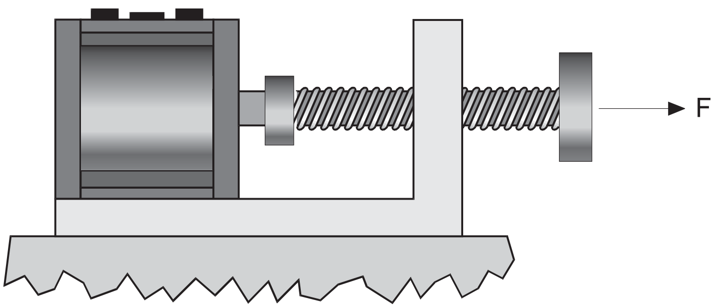
D = Outer diameter of wire or tube

d = Inside diameter of tube

Engineering

## Screw Clamping

Screw clamp with no acceleration, neglecting friction.




$$T_D = T_\alpha + T_f + T_L$$

$$T_\alpha = 0$$

$$T_f = 0$$ *see note below

$$T_L = \frac{Fp}{2\pi}$$

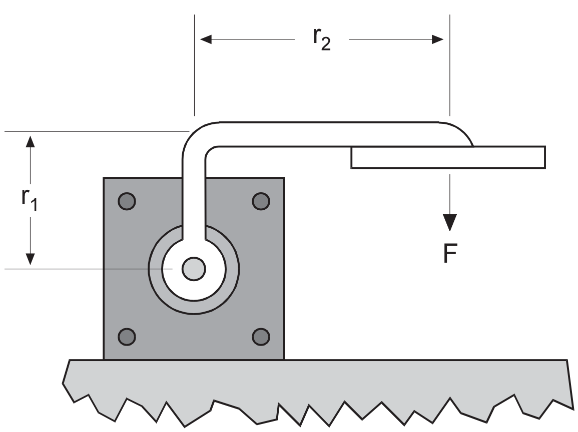
p = threads per inch

F = clamping force

*Friction torque could vary significantly based upon screw type, lubrication type, and bearings. To better approximate screw friction torque consult a machine design handbook.

## Simple Clamp

A simple clamping mechanism with no friction or acceleration.




$$T_D = T_\alpha + T_f + T_L$$

$$T_\alpha = 0$$

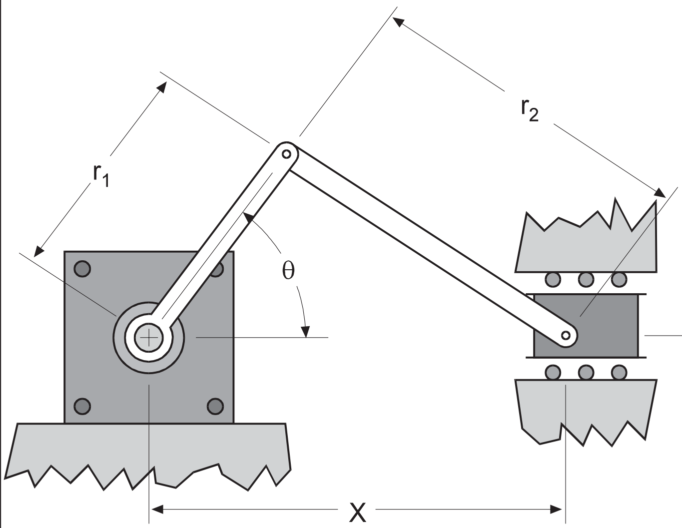
$$T_f = 0$$

$$T_L = Fr_2$$

This clamping mechanism may be suitable for holding down parts for assembly, but does not have the mechanical advantage or the linear motion provided in the next two examples.

## Linear Motion, Clamping

This type of clamp is characterized by a long stroke and a high clamping force as θ approaches zero.*




$$T_D = T_\alpha + T_f + T_L$$

$$T_\alpha = 0$$

$$T_f = 0$$

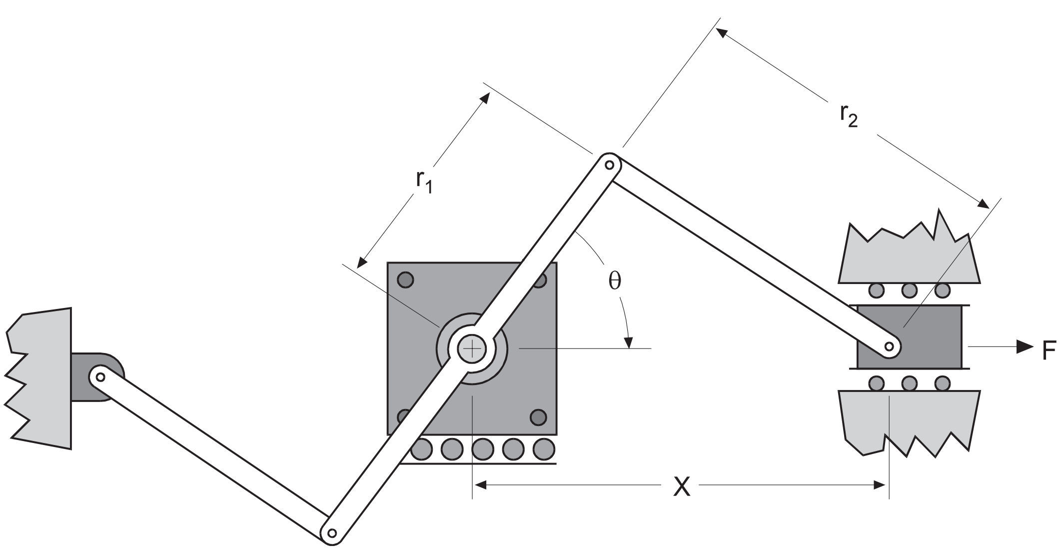
$$T_L = \frac{Fr_1}{2}\left[\frac{\sin 2\theta}{x - r_1 \sin\theta} + 2\sin\theta\right]$$

$$x = r_1 \cos\theta + \sqrt{r_2^2 - r_1^2 \sin^2\theta}$$

*To maintain control, θ should never be allowed to equal zero. Also, the force F should not be allowed to exceed the actuator's bearing capacity.

## Improved Linear Motion, Clamping

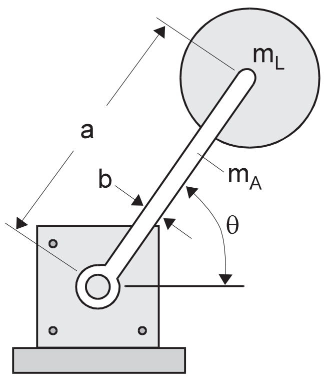
Same as above, except that force is not limited by the actuator's bearing capacity.




*To maintain control, θ should never be allowed to equal zero.

1800_engr.pmd, M&A

## Overcenter Load




If the mass $m_L$ is not free to rotate about its center, then its mass moment of inertia about its own center $I_L$ must be included in the equations for $T_a$ and $T_{a*}$ as follows:

$$T_\alpha = \left[\frac{1}{12} m_A(a^2 + b^2) + I_L + m_L a^2\right]\alpha$$

$$T_{\alpha^*} = \left[\frac{1}{12} m_A(a^2 + b^2) + I_L + m_L a^2\right]\alpha^*$$

The load is rotated in a vertical plane. Load torque $T_L$ is positive or negative depending upon position and direction of rotation. If the load resists the actuator's rotation, then $T_L$ is positive. The mass $m_L$ is free to rotate about its center.

$$T_D = T_\alpha + T_f + T_L$$

$$T_\alpha = I\alpha = \left[\frac{1}{12} m_A(a^2 + b^2) + m_L a^2\right]\alpha$$

$$T_f = r_b \mu_b (W_L + W_A)$$

$r_b$ = shaft bearing radius, not shown

$\mu_b$ = bearing coefficient of friction

In most cases $T_f$ will be very small compared to $T_L$.

$$T_L = \pm a \cos\theta \left[\frac{1}{2} W_A + W_L\right]$$

$$T_{D,MAX} = a\left[\frac{1}{2} W_A + W_L\right] + T_f + T_\alpha$$

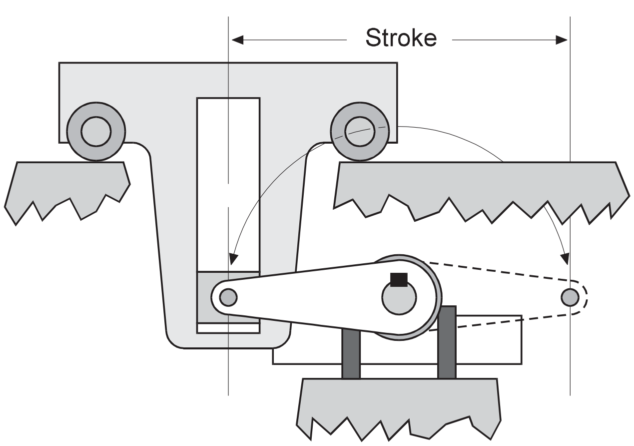
$$T_C = \frac{P_r V}{\theta} + T_{\alpha^*} - T_f \pm T_L$$

$$T_{C,MAX} = \frac{P_r V}{\theta} + T_{\alpha^*} - T_f + a\cos\theta\left(\frac{1}{2} W_A + W_L\right)$$

$$T_{\alpha^*} = I\alpha^* = \left[\frac{1}{12} m_A(a^2 + b^2) + m_L a^2\right]\alpha^*$$

$$P_C = T_C\left[\frac{\theta}{V}\right]$$

## Harmonic Drive




A harmonic linkage, as shown, is a compact and low cost method of providing linear motion with a very smooth acceleration.

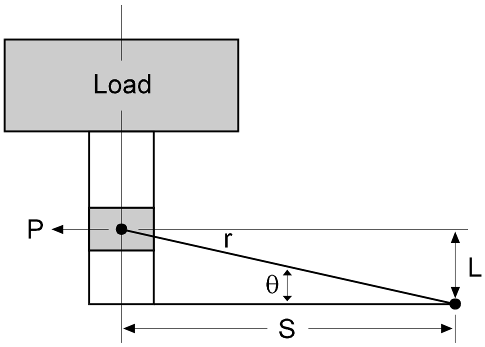
Flow controls can be adjusted to provide the smooth acceleration and deceleration necessary to handle fragile parts such as bottles or light bulbs.

The smooth acceleration and deceleration also enables optimum cycle times for handling automotive components on transfer lines.

A schematic model of the harmonic drive is shown on the following page. The equations for demand torque, $T_D$ will be defined in terms of the schematic model.

**continued . . .**

## Harmonic Drive, continued




$T_D$ = Actuator demand torque

r = Torque arm length

L = rsinθ

S = Stroke = rcosθ

W = Weight of the load

P = Bearing force against the slide

g = Gravitational acceleration

$\mu_L$ = Friction coefficient of the load

$\mu_S$ = Friction coefficient of the slide

θ = Actuator rotation in radians

ω = Angular velocity in rad/sec

t = time in seconds

A general equation can be derived relating the torque needed $T_D$ to the object weight W, radius r, friction coefficients $\mu_S$ and $\mu_L$, and the angular position θ. Because the position is expected to change with time t, the product of time and angular velocity ωt is substituted for θ in the equations below.

A.) $$\frac{T_D}{W} = \frac{r^2\omega^2}{g} \cos \omega t \sin \omega t + \frac{r^2\omega^2\mu_S}{g} \cos^2 \omega t + r\mu_L \sin \omega t + r\mu_L\mu_S \cos \omega t$$

Equation A gives the ratio of torque per weight for any time throughout the motion of the harmonic drive. To select an actuator it is the worst case torque that must be used for calculating $T_D$. The worst case, or maximum demand torque, can be calculated by taking the derivative of equation A with respect to time and solving for when the derivative is equal to zero. For reference the derivative of equation A with respect to time is given as equation B below.

B.) $$\frac{d(T_D/W)}{dt} = \frac{r\omega^2}{g}(2\cos^2 \omega t - 1) - \frac{2r\omega^2\mu_S}{g} \cos \omega t \sin \omega t + \mu_L \cos \omega t - \mu_L\mu_S \sin \omega t$$

Equations A and B have been solved to create the graphs on the next page for coefficients of friction of 0.05 and 0.25 respectively. (For the graphs, the coefficients $\mu_S$ and $\mu_L$ are set equal to each other.)

The preceding equations assume a constant angular velocity ω; as the inertia from the moving load tends to drive the actuator during the deceleration phase, it is recommended that pressure compensated restrictor type flow controls be used.

C.) $$T_C = \frac{P_r V}{\theta} + T_D - W\left[\frac{T_D}{W} \text{ (evaluated at 10 second throw time on graph)}\right]$$

Where $P_r$ is the relief valve pressure, V is the actuator displacement, and θ is the actuators rated rotation in radians.

Engineering

## Harmonic Drive, continued

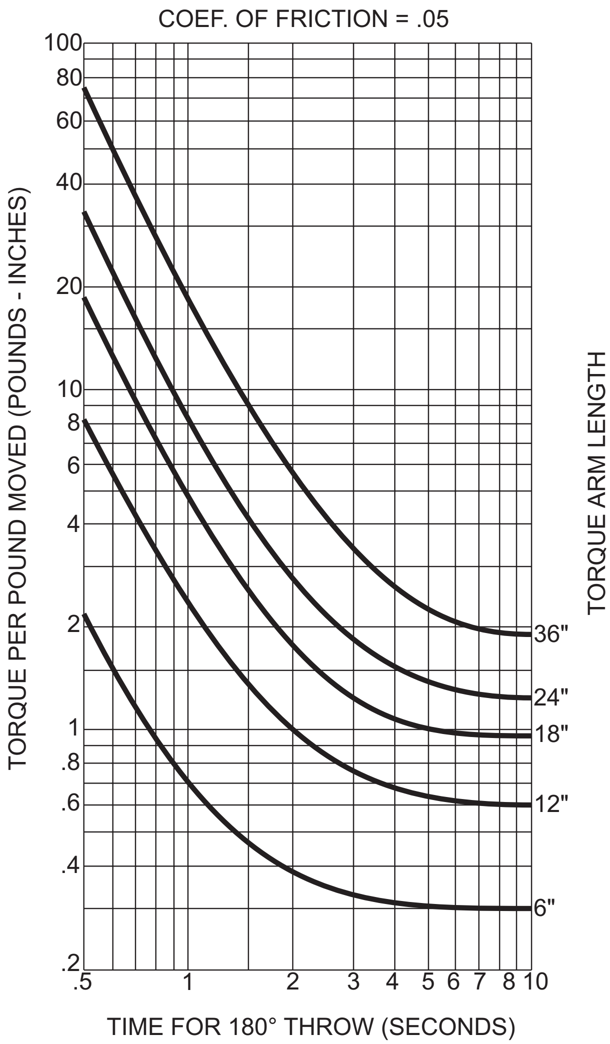



*Figure 9*

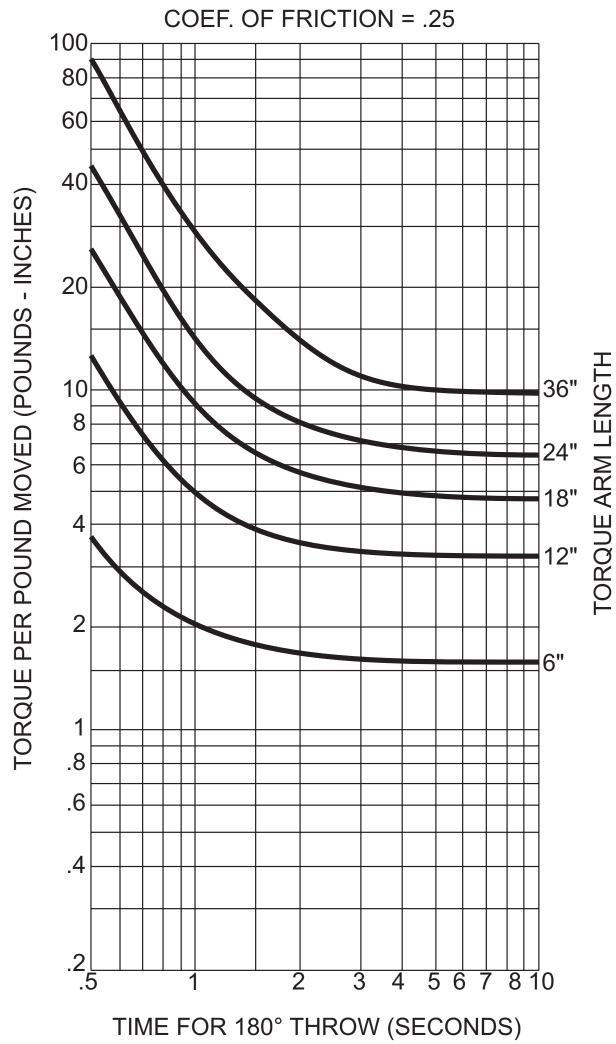



*Figure 10*

**Example1E:** A 400 lb block must be moved 48 inches in 4 seconds with a harmonic drive. The block and the slide are supported with roller bearings (coeff. of friction = 0.05) a.) Calculate the demand torque $T_D$ required. b.) Calculate the cushion torque if a 180° rotary actuator with a displacement V of 7.6 in³ is chosen, and the relief valve pressure $P_r$ is set at 500 psi.

**Solution:**

a.) Use the curves for coef. of friction = 0.05 and draw a line from the 4 seconds on the time axis to intersect with the 24 inch torque arm curve.

$$\frac{\text{48 in travel}}{2} = \text{24 in torque arm length}$$

The intersecting line shows a requirement for $T_D$/W = 1.5: $T_D = 1.5\ (400\ lb) = 600\ lb\text{-}in$ **answer**

b.) Use equation C from the previous page to calculate the cushion torque $T_C$:

$$T_C = \frac{500\ psi\ (7.6\ in^3)}{\pi} + 600\ lb\text{-}in - \left(\frac{1.2\ lb\text{-}in}{lb}\right)(400\ lb) = 1{,}329\ lb\text{-}in$$ **answer**

## Sizing Rotary Actuators

The smallest rotary actuator displacement, that can be used in an application, is that displacement which can both deliver sufficient torque to move the load and also withstand the pressure required to stop the load. (Recall that cushion torque is generated by back pressure that is often greater than system pressure.) A method of determining the smallest rotary actuator displacement is summarized in Fig. 11 and outlined step-by-step below. (Also see the example problem on page 23.)

**Note:** This method is for constant torque rotary actuators such as vane, rack and pinion, or helical styles.

A. Determine the maximum allowable safe system pressure $P_r$ that the pump and components can tolerate. This is typically the highest pressure the pump can supply to the system; however, this is not the actual system working pressure. The actual working pressure is determined after an actuator is selected.

B. Calculate the demand torque required. The demand torque $T_D$ is given by equation 3 repeated here:

$$T_D = T_\alpha + T_f + T_L$$

Definitions for the above torque components and examples for calculation of $T_D$ were discussed previously under the heading "Calculating Torque Requirements."

C. Calculate V/θ based upon $T_D$ and the maximum system pressure chosen in Step A. V/θ is the volume displacement per one radian of rotation for a rotary actuator. V/θ can be calculated from the Equation 7, or by using Fig. 14. (Fig. 12 for S.I. units).

Equation 7) $$\frac{V}{\theta} = \frac{T_D}{P_r}$$

D. Calculate the cushion torque $T_C$ required. In any application where the actuator has cushions, a deceleration valve, or any form of meter-out flow control, the flow out of the actuator is restricted creating a back pressure on the outlet side of the actuator. This back pressure is what creates the cushion torque which acts to decelerate, or cushion the actuator as it approaches the end of its rotation. The cushion torque can be calculated by the methods presented under the heading "Calculating Torque Requirements."

E. Calculate the cushion pressure $P_C$, for the rotary actuator with V/θ as calculated in step C, and $T_C$ as calculated in step D. Use Equation 8:

Equation 8) $$P_C = T_C \left[\frac{\theta}{V}\right]$$

**Note: Equation 8 calculates the *average* back pressure $P_C$, *not* the *maximum* back pressure. In most cases back pressure will not be constant and will exceed the average value calculated here.**

F. If the value for $P_C$ is considered impractical for your application, some ways to lower it are:

1. Reduce system pressure, then recalculate Steps C through E.
2. Increase the time for deceleration, then recalculate Steps D and E.
3. Use an external shock absorber.

G. Calculate the needed displacement V for the rotation you need. Use Equation 9 or Fig. 15. (Fig. 13 for S.I. units.)

Equation 9) $$V = \theta \left[\frac{V}{\theta}\right]$$

H. Select a rotary actuator with a torque rating greater than $T_D$, a displacement greater than V, and a pressure rating greater than $P_C$. Calculate system operating pressure P based upon the selected actuator's rated torque T, and the selected actuator's V/θ value using Equation 10 or Fig. 12.

Equation 10) $$P = T \left[\frac{V}{\theta}\right]$$

The relief valve setting $P_r$ must be less than the maximum pressure from Step A, must be greater than P calculated in Equation 10, must not exceed the actuator's rated working pressure, and must be high enough to compensate for pressure drop through valves and lines.

## Actuator Flow Chart

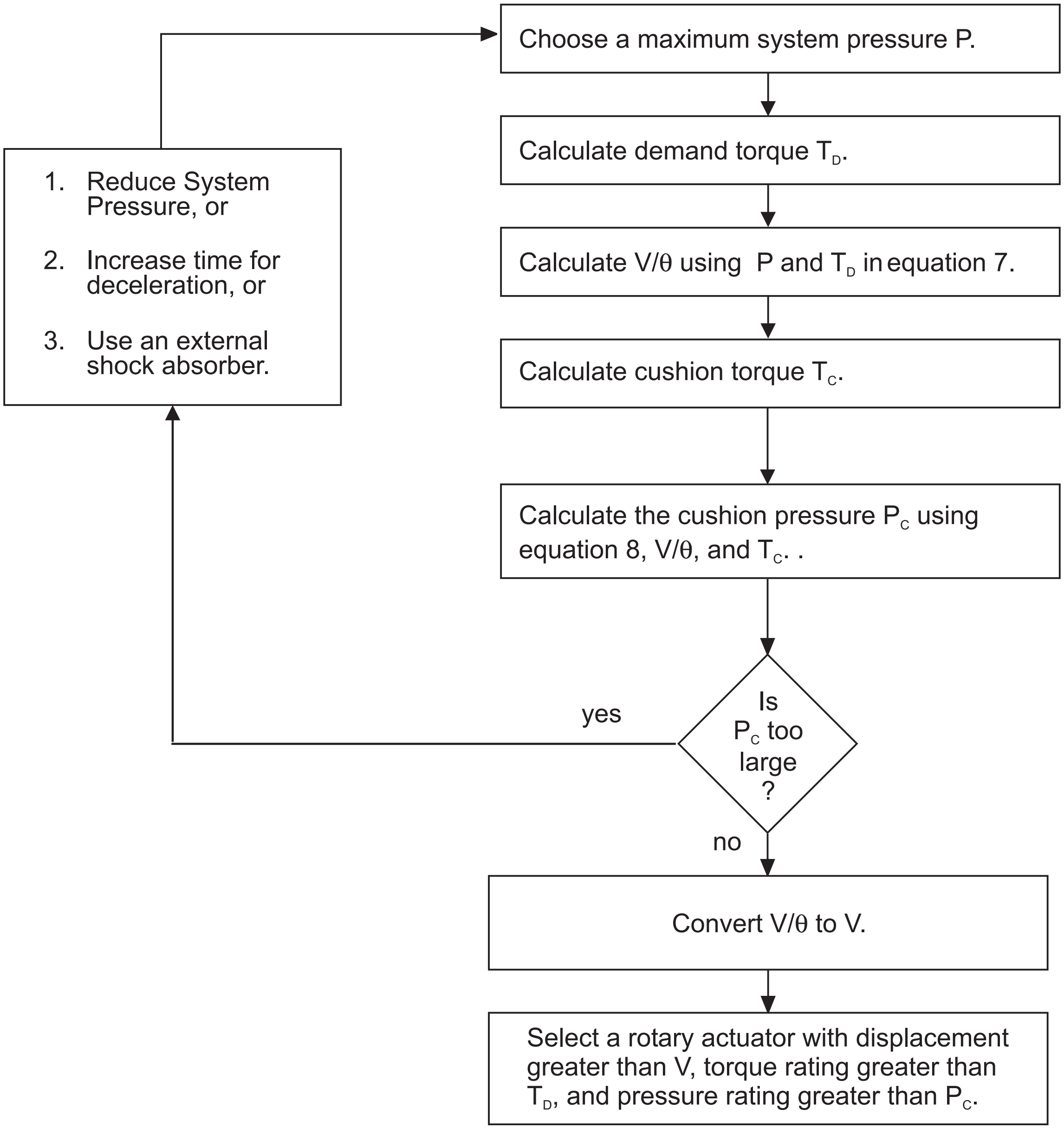



*Figure 11: Flow chart for selecting a minimum volume actuator. See example problem page 23.*

## Minimum Possible Rotary Actuator Volume (S.I. Units)

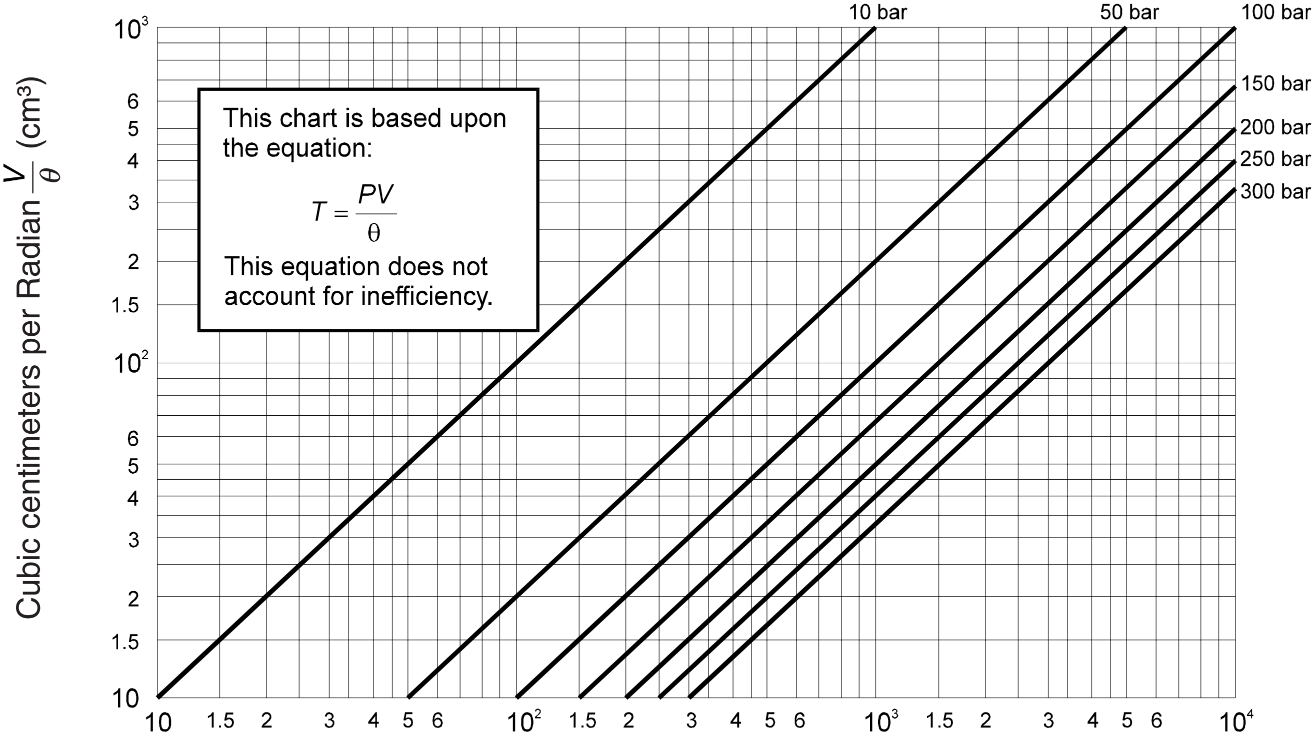



*Figure 12*

## Coverting Volume per Radian to Volume Displacement (S.I. Units)

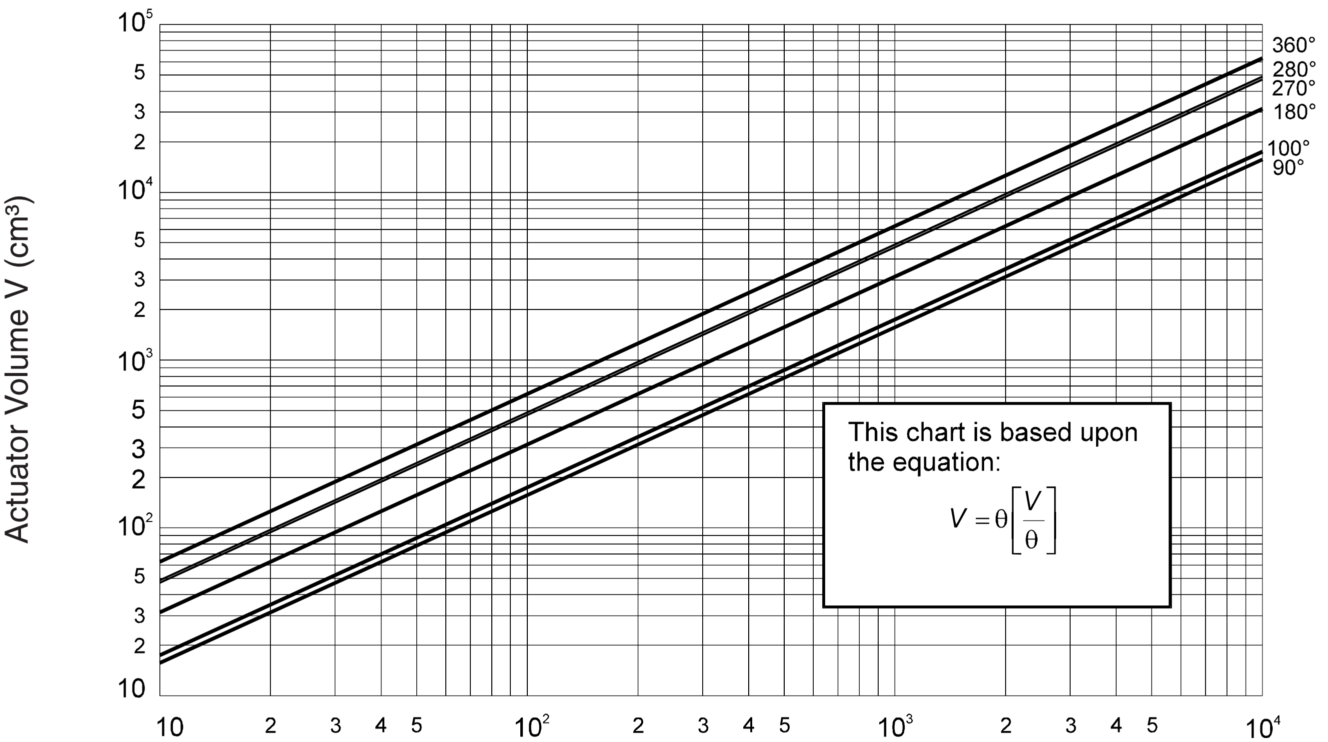



*Figure 13*

Engineering

## Minimum Possible Rotary Actuator Volume (English Units)

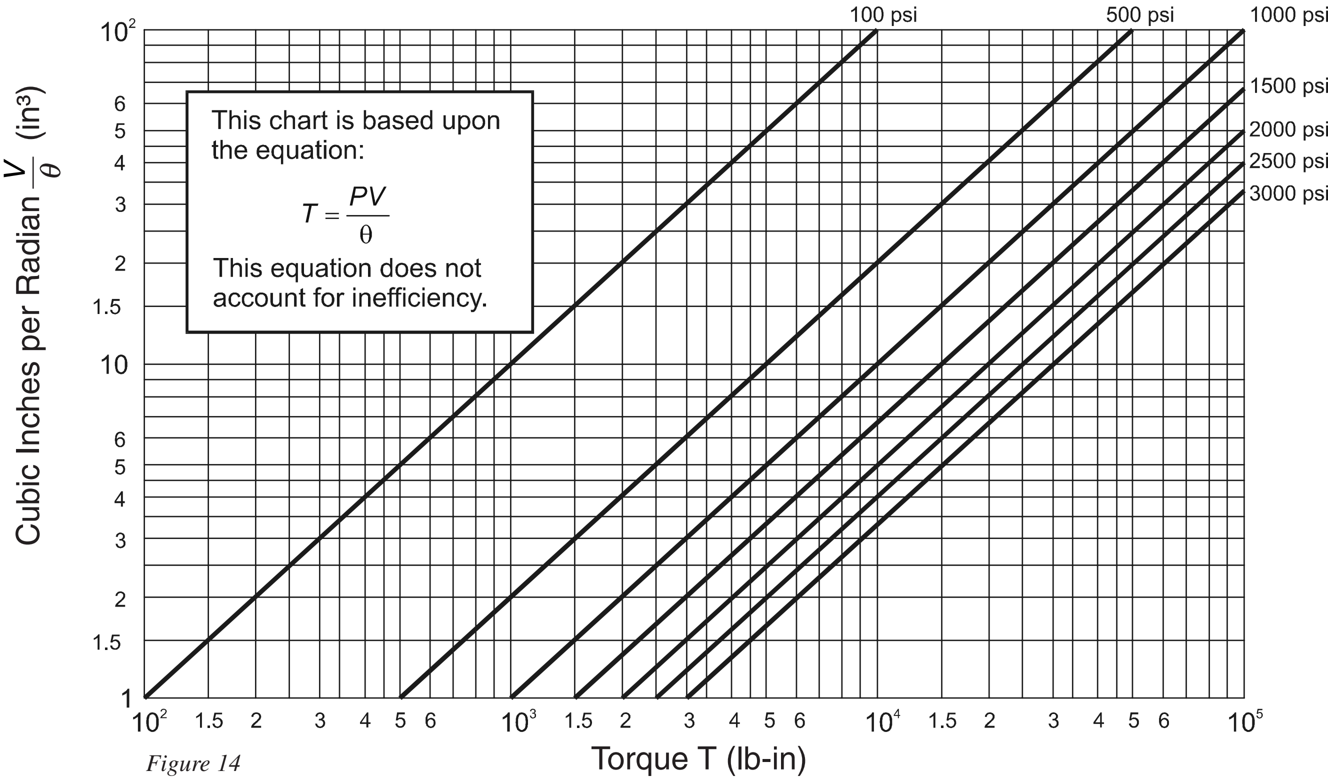



*Figure 14*

## Coverting Volume per Radian to Volume Displacement (English Units)

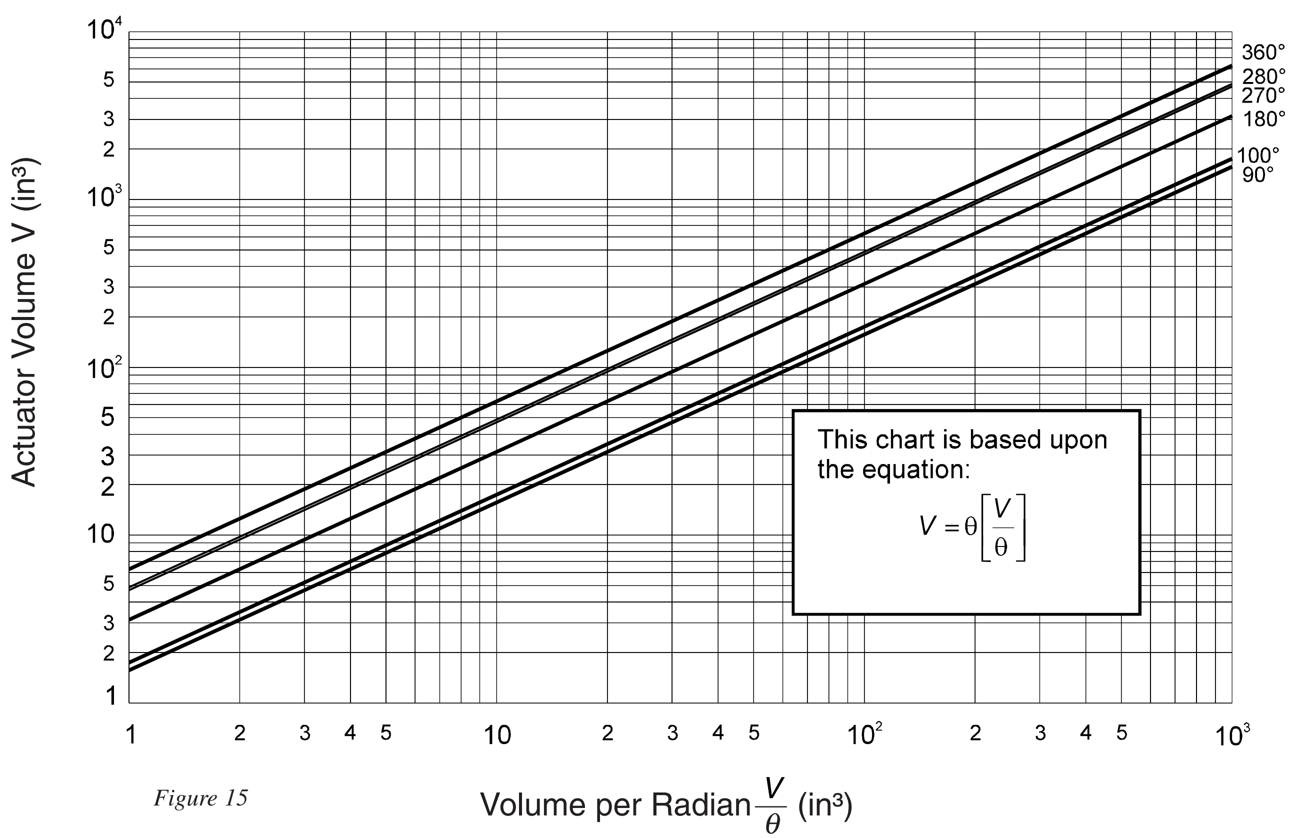
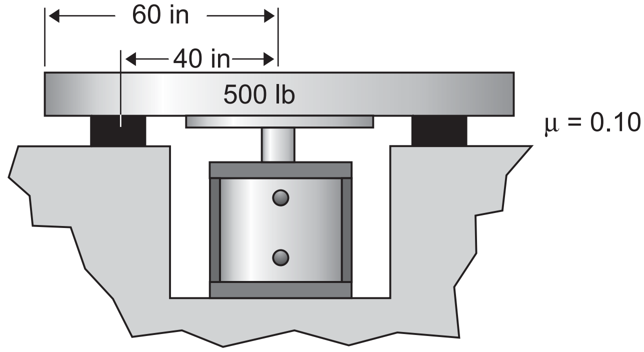



*Figure 15*




**Example 2E:** For the rotary index table shown , calculate the minimum requirements for an actuator to turn the index table 180°. Use $\alpha = \alpha^* = 2.5$ rad/sec², $P_r = 1000$ psi.

**Solution:** Use the flow chart Fig. 11, on page 20.

A.) The supply pressure $P_r$ is given as 1000 psi.

B.) Calculate the demand torque $T_D$:

$$T_D = T_\alpha + T_f + T_L$$

$$T_\alpha = I\alpha$$

$$I = \frac{1}{2}mr^2 = \frac{1}{2}\left(\frac{500\ lb}{386\ in/sec^2}\right)(60\ in)^2 = 2332\ lb\text{-}in\text{-}sec^2$$

$$\alpha = 2.5\ rad/sec^2$$

$$T_\alpha = (2332\ lb\text{-}in\text{-}sec^2)(2.5\ rad/sec^2) = 5{,}830\ lb\text{-}in$$

$$T_f = \mu W r_b = (0.10)(500\ lb)(40\ in) = 2{,}000\ lb\text{-}in$$

$$T_L = 0$$

$$T_D = 5{,}830\ lb\text{-}in + 2{,}000\ lb\text{-}in + 0 = 7{,}830\ lb\text{-}in$$

C.) Calculate $\frac{V}{\theta}$ use Equation 7 or Fig. 14:

$$\frac{V}{\theta} = \frac{T_D}{P_r} = \frac{7{,}830\ lb\text{-}in}{1000\ psi} = 7.83\ in^3$$

D.) Calculate cushion torque $T_C$ (see equations for Rotary Index Table)

$$T_C = \frac{P_r V}{\theta} + T_{\alpha^*} - T_f + T_L = 1000\ psi\,(7.83\ in^3) + 5{,}830\ lb\text{-}in - 2000\ lb\text{-}in + 0 = 11{,}660\ lb\text{-}in$$

Notice that since the rate of deceleration $\alpha^*$ is the same as the rate of acceleration $\alpha$, the deceleration torque $T_{\alpha^*}$ is the same as the acceleration torque $T_\alpha$.

E.) Calculate the cushion pressure $P_C$:

$$P_C = T_C\left[\frac{\theta}{V}\right] = \frac{11{,}660\ lb\text{-}in}{7.83\ in^3} = 1{,}489\ psi$$

F.) Is the cushion pressure too high? The cushion pressure (1,489 psi) is within the capability of most hydraulic rotary actuators. In cases where $P_C$ becomes the dominant selection criteria, the designer should consider repeating the sizing calculations with a lower value for $P_r$, decreasing the deceleration rate $\alpha^*$, or using an external shock absorber.

G.) Calculate the displacement V for the 180° rotation. Use Equation 9 or Fig. 15.

$$V = \theta\left[\frac{V}{\theta}\right] = \pi(7.83\ in^3) = 24.6\ in^3$$

H.) Select a rotary actuator with a torque rating greater than 7,830 lb-in with a supply pressure of 1000 psi, it will have a displacement greater than 24.6 in³, and must have a maximum pressure rating over 1,489 psi.

## TABLE 4: CONVERT Vs to V

Based upon equation $V_S\theta = V$ for converting specific volume into volume.

## ACTUATOR ROTATION VOLUME

| | ACTUATOR ROTATION (DEGREES) | | | | | |
|---|---|---|---|---|---|---|
| **Vs** | **90** | **100** | **180** | **270** | **280** | **360** |
| 10 | 15.71 | 17.45 | 31.42 | 47.13 | 48.86 | 62.84 |
| 11 | 17.28 | 19.20 | 34.56 | 51.84 | 53.75 | 69.12 |
| 12 | 18.85 | 20.94 | 37.70 | 56.56 5 | 8.63 | 75.41 |
| 13 | 20.42 | 22.69 | 40.84 | 61.27 | 63.52 | 81.69 |
| 14 | 21.99 | 24.43 | 43.98 | 65.98 | 68.40 | 87.98 |
| 15 | 23.57 | 26.18 | 47.13 | 70.70 | 73.29 | 94.26 |
| 16 | 25.14 | 27.92 | 50.27 | 75.41 | 78.18 | 100.5 |
| 17 | 26.71 | 29.67 | 53.41 | 80.12 | 83.06 | 106.8 |
| 18 | 28.28 | 31.41 | 56.55 | 84.83 | 87.95 | 113.1 |
| 19 | 29.85 | 33.16 | 59.70 | 89.55 | 92.83 | 119.4 |
| 20 | 31.42 | 34.90 | 62.84 | 94.26 | 97.72 | 125.7 |
| 22 | 34.56 | 38.39 | 69.12 | 103.7 | 107.5 | 138.2 |
| 24 | 37.70 | 41.88 | 75.41 | 113.1 | 117.3 | 150.8 |
| 26 | 40.84 | 45.37 | 81.69 | 122.5 | 127.0 | 163.4 |
| 28 | 43.99 | 48.86 | 87.98 | 132.0 | 136.8 | 176.0 |
| 30 | 47.13 | 52.35 | 94.26 | 141.4 | 146.6 | 188.5 |
| 32 | 50.27 | 55.84 | 100.5 | 150.8 | 156.4 | 201.1 |
| 34 | 53.41 | 59.33 | 106.8 | 160.2 | 166.1 | 213.7 |
| 36 | 56.56 | 62.82 | 113.1 | 169.7 | 175.9 | 226.2 |
| 38 | 59.70 | 66.31 | 119.4 | 179.1 | 185.7 | 238.8 |
| 40 | 62.84 | 69.80 | 125.7 | 188.5 | 195.4 | 251.4 |
| 42 | 65.98 | 73.29 | 132.0 | 197.9 | 205.2 | 263.9 |
| 44 | 69.12 | 76.78 | 138.2 | 207.4 | 215.0 | 276.5 |
| 46 | 72.27 | 80.27 | 144.5 | 216.8 | 224.8 | 289.1 |
| 48 | 75.41 | 83.76 | 150.8 | 226.2 | 234.5 | 301.6 |
| 50 | 78.55 | 87.25 | 157.1 | 235.7 | 244.3 | 314.2 |
| 55 | 86.41 | 95.98 | 172.8 | 259.2 | 268.7 | 345.6 |
| 60 | 94.26 | 104.7 | 188.5 | 282.8 | 293.2 | 377.0 |
| 65 | 102.12 | 113.4 | 204.2 | 306.3 | 317.6 | 408.5 |
| 70 | 109.97 | 122.2 | 219.9 | 329.9 | 342.0 | 439.9 |
| 75 | 117.83 | 130.9 | 235.7 | 353.5 | 366.5 | 471.3 |
| 80 | 125.68 | 139.6 | 251.4 | 377.0 | 390.9 | 502.7 |
| 85 | 133.54 | 148.3 | 267.1 | 400.6 | 415.3 | 534.1 |
| 90 | 141.39 | 157.1 | 282.8 | 424.2 | 439.7 | 565.6 |
| 95 | 149.25 | 165.8 | 298.5 | 447.7 | 464.2 | 597.0 |
| 100 | 157.1 | 174.5 | 314.2 | 471.3 | 488.6 | 628.4 |


**Parker Hannifin Corporation**
Actuator Division
Wadsworth, Ohio USA

## Calculating Required Pump Flow

The flow rate required for a rotary actuator can be determined by the desired time for rotation and the rotary actuator's displacement. This is shown in Equation 5.

The equation is also plotted as Figure 16.

$$Q = \frac{V}{t}$$

where Q = Flow rate
V = Rotary actuator displacement
t = Time to fill displacement

**EXAMPLE:** A 280° vane rotary actuator is chosen to provide a 194° rotation in 2 seconds. If the rotary actuator's displacement is 77.8 in$^3$, find what flow rate is required from the pump. Assume constant angular velocity.

**SOLUTION:** The actuator is only rotating 194°, so the volume of oil required for this rotation is:

$$V = 77.8\ in^3\ (194/280)$$

$$V = 53.9\ in^3\ for\ 194°\ rotation$$

$$Q = \frac{V}{t}$$

$$Q = \frac{59.3\ in^3\ (60\ sec/min)}{2\ sec\ (231\ in^3/gal)} = 7\ \text{GPM}$$ **ANSWER**

**EXAMPLE:** A 180° rack and pinion rotary actuator is to accelerate from 0 to some angular velocity w during its first 10° of rotation, then remain at that angular velocity for the next 150° of rotation, then decelerate back to 0 radians/sec during the last 20°. The actuator is to rotate the total 180° in less than 2 seconds. If the actuator's displacement is 36 in$^3$, find:

A. The angular velocity w after the first 10° of rotation
B. The pump flow rate required for the rotary actuator
C. The pump flow required if the actuator traveled the entire 180° in 2 seconds at a constant angular velocity

**SOLUTION:**

A. Assume constant acceleration during the first 10° and constant deceleration during the last 20°.

$$2\ sec = t_1 + t_2 + t_3$$

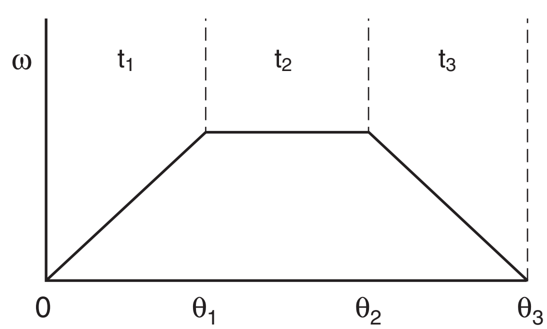
$$t_1 = 2\frac{\theta_1 - 0}{\omega} = \frac{2}{\omega}(10°)\frac{\pi}{180°} = \frac{1}{\omega}(.35)$$

$$t_2 = \frac{\theta_2 - \theta_1}{\omega} = \frac{1}{\omega}(150°)\frac{\pi}{180°} = \frac{1}{\omega}(2.62)$$

$$t_3 = 2\frac{\theta_3 - \theta_2}{\omega} = \frac{2}{\omega}(20°)\frac{\pi}{180°} = \frac{1}{\omega}(.70)$$

$$2\ sec = \frac{1}{\omega}[.35 + 2.62 + .70]$$

$$\omega = 1.83\ \text{rad/sec}$$ **ANSWER**




**CONTINUED ON NEXT PAGE**

**EXAMPLE CONTINUED**

B. 180° rotary actuator has a volume displacement of 36 in3.
The cubin inches per radian can be expressed as:

$$V_s = \frac{V}{\theta} = \frac{36\ in^3}{\pi}\ radians$$

$$V_s = 11.5\ in^3/radian$$

NOTE: 180° = p radians

The actuator must be able to rotate at 1.83 rad/sec so the pump flow must be:

$$Q = V_s\omega$$

$$Q = (11.5\ in^3/radian)\ (1.83\ rad/sec)\ \frac{1\ gal}{231\ in^3}\ \frac{60\ sec}{min}$$

$Q$ = 5.5 GPM **ANSWER**

C. If the entire 180° were traversed at constant speed in 2 seconds, the pump flow would be:

$$Q = \frac{V}{t}$$

$$Q = \frac{38\ in^3}{2\ sec}\ \frac{1\ gal}{231\ in^3}\ \frac{60\ sec}{min}$$

$Q$ = 4.7 GPM **ANSWER**

Notice that in the above example, it is necessary to take into account the time required for acceleration and deceleration of the actuator in order to determine the maximum velocity required. It is the maximum velocity of the actuator that will determine the maximum flow required. Equations for velocity and acceleration are provided on page 10 in Table 5.

## Time/Revolution vs. Volume

Based on 100% efficiency and the equation:

$t = .26\ V/Q$

where $t$ = time/rev. in seconds

$V$ = displacement in $in^3$

$Q$ = oil flow in GPM

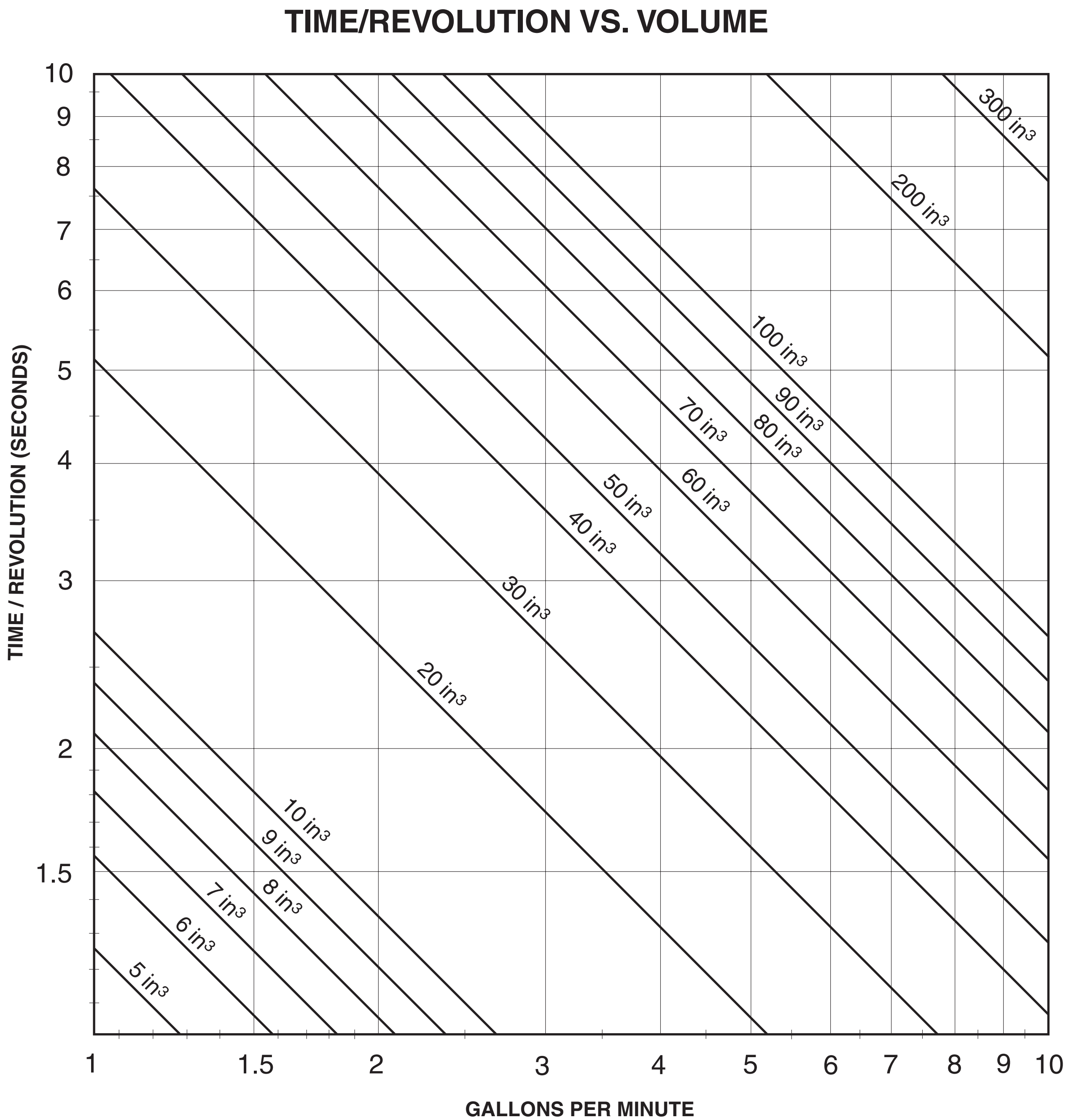



*Figure 16*

## Composite Hydraulic Circuit

When designing hydraulic operating circuits for rotary actuators, consideration should be given to the following criteria:

1. actuator rotational velocity
2. kinetic energy developed
3. actuator holding requirement
4. system filtration

Figure 17 is a composite drawing showing general recommendations for sample circuitry. It is intended as a guide only. Flow control valves (1) in the meter-out position provide controlled actuator velocity. Care should be taken if the load moves overcenter, as the combination of load and pump generated pressure may exceed the actuator rating.

To protect the actuator and other system components from shock pressures caused when the actuator is suddenly stopped in mid-stroke, crossover relief valves (2) should be installed as close to the actuator as possible. These relief valves also protect the actuator and system if the load increases and "back drives" the hydraulic system.

In applications involving high speeds or heavy loads, the built up kinetic energy may be too much for cushions to absorb during their 20° of operation. By using cam or lever operated deceleration valves (3) the deceleration arc can be increased beyond 20° so that kinetic energy can be absorbed more gradually and without overpressuring the actuator. Where there is a need to hold the load in intermediate positions for extended periods of time, pilot operated check valves (4) should be used. These must be used with leakproof actuator seals to hold the load in position; any bypass flow allows eventual drifting of the load.

**Warning: For safety reasons, some applications require a mechanical locking device for holding loads over an extended period of time.**

As with most standard hydraulic circuits, rotary actuator applications should have filtration to provide a continuous cleanliness rating of no more than 390 particles greater than 10 micron per milliliter of fluid. This is an ISO 17/14 fluid cleanliness classification. Filters (5) should be fitted and maintained to ensure this minimum level.

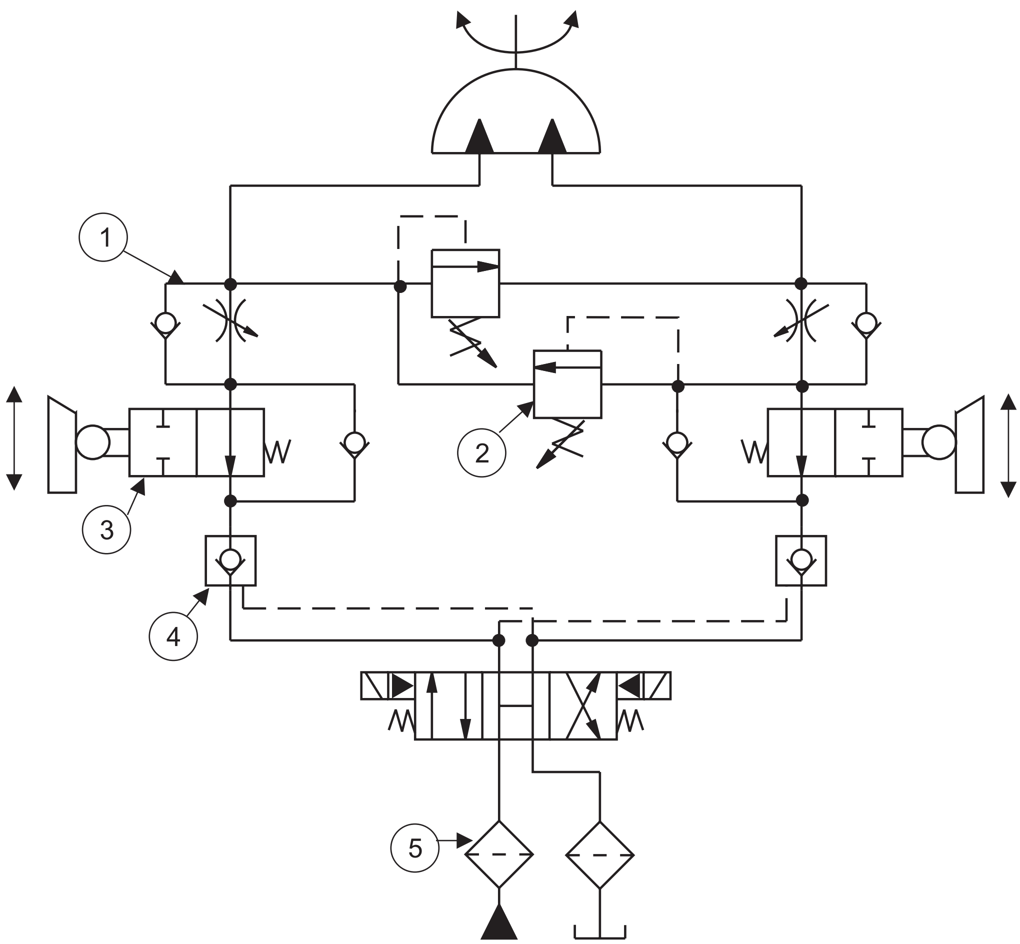



*Figure 17 Hydraulic rotary actuator circuit*

## Electrohydraulic Circuitry

The use of electrohydraulic components for rotary actuator applications can provide greater system flexibility. Figure 18 is a representative circuit showing some possible applications of electrohydraulic valves. Proportional or servo control valves (1) can provide continuous position, velocity or acceleration control of loads, and "closing the loop" around a position feedback device and can provide even greater control and velocity profiles for overcenter or varying loads. Even more precise position control is possible with the use of vane actuators or anti-backlash devices on rack and pinion units.

Torque control can also be achieved with servovalve (1) by taking advantage of the valve's pressure gain region.

All of the considerations from the composite hydraulic circuit (Fig. 17) are still relevant. Crossover relief valves (3) should be installed if there are uncontrolled sudden stops in mid-stroke, and caution should be exercised when running overcenter, running with high speeds, or moving high inertia loads. Filtration (4) is still a consideration, but the actuator requirements (ISO 17/14 class) are usually less demanding than the filtration requirements of today's electrohydraulic control valves.

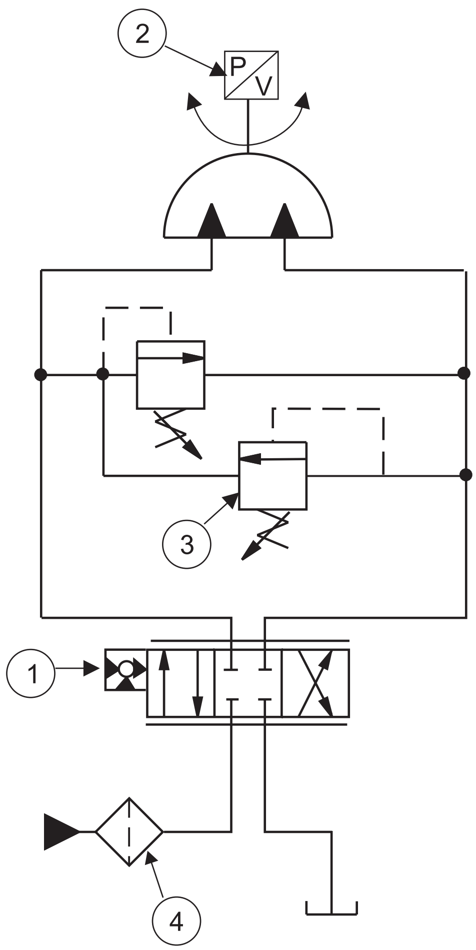



*Figure 18 Electrohydraulic rotary actuator circuit*

Engineering

## Installation Instructions and Options

### A. Stops

Vane units should not use the vane and stator as a positive stop. For light to medium loads, an internal taper lock stop, or external stops mounted to the machine framework, may be used. For high inertia or high speed loads, externally mounted valving or deceleration devices should be used to minimize system shocks.

Rack and pinion units can be stopped at the end of stroke provided the loads and speeds are not too high. Cushions can be used to decelerate the load to a gentle stop, providing the maximum actuator pressure rating is not exceeded by the cushions. Again, for high inertia or high speed loads, externally mounted valving or deceleration devices should be used to minimize system shocks.

### B. Surge Pressures

Surge or shock pressure in excess of the actuator rated pressure are detrimental to unit life and must be avoided. Crossover relief valves mounted adjacent to the actuator can help reduce these abnormal pressure peaks.

Pressure developed by cushion or deceleration valves should also be kept below rated pressure.

### C. Angular Velocity

Angular velocity can be controlled by metering the flow into or out of the actuator ports. This can be accomplished by the use of flow control valves; or if more sophisticated control is required, through the use of proportional or servo valves.

Care should be taken when using a meter-out circuit if the load moves over center, as the combination of load and pump generated pressures may exceed the actuator rating.

### D. Drains

Some actuators are fitted with drain ports to minimize external leakage possibilities. These drain ports should be connected directly back to the oil reservoir with a minimum of back pressure (3.5 bar / *50 psi* maximum).

### E. Gear Chamber

Some rack and pinion actuators are supplied with the gear chambers filled with a molybdenum disulfide grease to better absorb gear stress and extend gear life. This chamber should be checked and filled periodically to ensure adequate gear lubrication. The housing can be fitted with a small relief valve that vents excess pressure in the gear chamber to the atmosphere. This is an indication of pressure seal wear, because high pressure oil is bypassing the piston seal and pressuring the gear chamber. The piston seals should then be replaced.

### F. Fluid Medium and Seals

For hydraulic usage, a clean, filtered, high-quality mineral-based hydraulic fluid with 150 to 500 S.U.S. viscosity at 100°F is recommended for use with standard Buna N seals. Cleanliness should be maintained to an ISO code 17/14 level.

Standard seal compound is Buna N for mineral-based hydraulic fluid. Other seal materials can be provided for most operating fluids. If there is a question about the correct seal compound, provide the name and type of operating fluid to the actuator manufacturer and ask for their recommendation.

### G. Shaft Couplings

Couplings should engage the full length of the shaft keyway and pressure should only be applied after support has been provided on the opposite end of the shaft. Shafts should be within 0.005 TIR to ensure proper alignment.